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Office hour: 3:30 ~ 4:30pm Monday

HH 401

Preliminary:

Compound Interest: $A(t) = A(0)(1+i)^t$.

Simple Interest: $A(t) = A(0)(1+it)$

Present value: $A(0) = A(t)v^t$. $v = \frac{1}{1+i}$ is present value factor.

Treasury Bills: price $= \frac{\text{face amount}}{1+it}$

1.1.2.

how to measure

(a) $A(0) = 2500$, $t = \frac{10}{1} = 10$, $i = 4\%$.

$A(t) = A(0)(1+it) = 2500 \times (1+4\% \times 10) = 3500$.

(b) $A(t) = A(0)(1+i)^t = 2500 \times (1+4\%)^{10} = 3700.61$

(c) $A(0) = 2500$, $t = \frac{10}{0.5}$, $i = 2\%$,

$A(t) = A(0)(1+i)^t = 2500 \times (1+2\%)^{\frac{10}{0.5}} = 3714.87$

(d) $t = \frac{10}{0.25}$, $i = 1\%$,

$A(t) = A(0)(1+i)^t = 2500 \times (1+1\%)^{\frac{10}{0.25}} = 3722.16$

1.1.10.

(a) $A(0) = 1000$, $A(t) = 3000$, $i = 12\%$,

$(1+i)^t = \frac{A(t)}{A(0)} \Rightarrow t\log(1+i) = \log A(t) - \log A(0) \Rightarrow t = \frac{\log A(t) - \log A(0)}{\log(1+i)} = 9.694$.

(b) Integer part : 9.

$A(0)(1+i)^{[t]}(1+iS) = 1000(1+12\%)^9(1+12\% \times S) = 3000 \Rightarrow S = 0.6819.$

$t = 9 + 0.6819 = 9.6819.$

(c)
It's monthly $i = 1\%$, $1000(1+0.01)^t = 3000 \Rightarrow t = 110.41$ months.

(d) $(1+i)^t = \frac{A(t)}{A(0)} \Rightarrow i = \sqrt[t]{\left(\frac{A(t)}{A(0)}\right)} - 1 = \sqrt[10]{\frac{3000}{1000}} - 1 = 0.1161$

(e) $j = \sqrt[t]{\frac{A(t)}{A(0)}} - 1 = \sqrt[120]{\frac{A(t)}{A(0)}} - 1 = 0.009197.$

1.2.1.

$$\begin{array}{ccccc} & 1 & 2 & 3 & 4 \\ 0 & 5000 & 5000 & 5000 & 5000 \end{array}$$

$v = \frac{1}{1+i} = \frac{1}{1+6\%}$

present value $= 5000v + 5000v^2 + 5000v^3 + 5000v^4 = 17,325.53$

1.2.4.

$v_{6\%} = \frac{1}{1+6\%}$. $v_{7\%} = \frac{1}{1+7\%}$ $v_{9\%} = \frac{1}{1+9\%}$.

$PV = 1000\, v_{6\%}^3\, v_{7\%}^4\, v_{9\%}^3 = 494.62$

1.2.15.

(a) $t = \frac{182}{365}$. $P = \frac{\text{face amount}}{1+it} = \frac{100,000}{1+10\% \times \frac{182}{365}} = 95,250.52.$

(b) $P = \frac{F}{1+it}$, $\frac{dP}{di} = -\frac{F}{(1+it)^2} \cdot t$, $\frac{dP}{di} \doteq \frac{P(i+\Delta i) - P(i)}{\Delta i} = \frac{\Delta P}{\Delta i}$, $\Delta i = 10.1\% - 10\%$.

$\Rightarrow \Delta P = \frac{dP}{di} \cdot \Delta i = -23733.34$

(c) $t = \frac{91}{365}$ when $t = 0$. $\frac{dP}{di} = -\frac{F}{(1+t)^2} \cdot t = 0$